

MATHEMATICS

Chapter 13: Surface Areas and Volumes

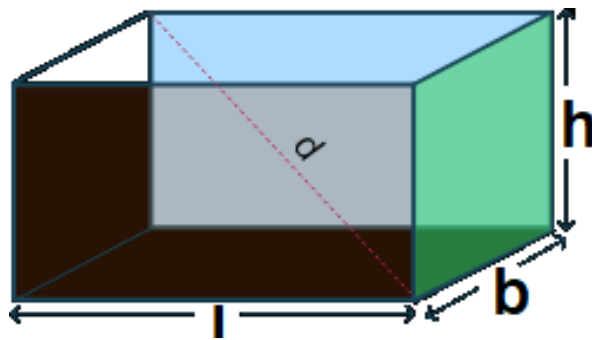


Surface Areas and Volumes

1. **Surface area** of a solid is the sum of the areas of all its faces.
2. The space occupied by a solid object is the **volume** of that object.
3. If l , b , h denote respectively the length, breadth and height of a **cuboid**, then: Lateral surface area or Area of four walls = $2(l + b)h$

Total surface area = $2(lb + bh + hl)$ Volume = $l \times b \times h$

Diagonal of a cuboid =

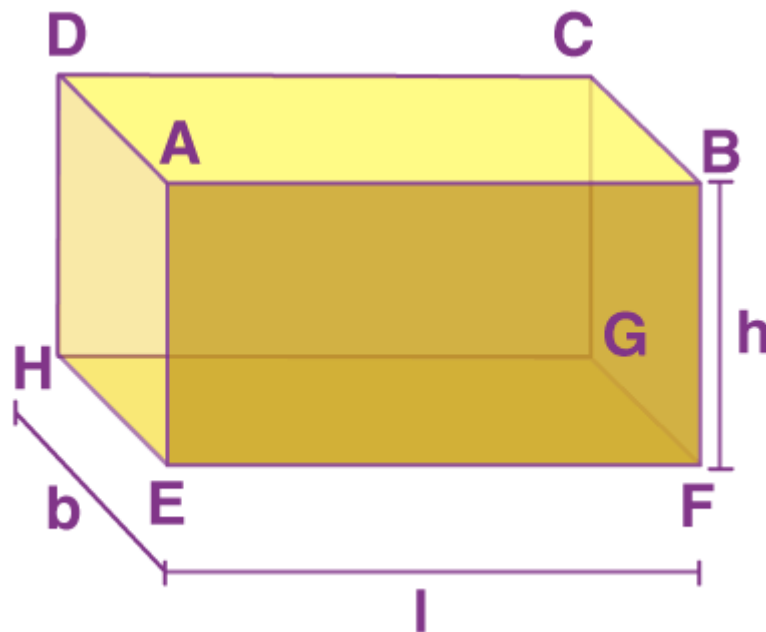


Surface Area and Volume of Cuboid

A cuboid is the region covered by its six rectangular faces. The surface area of a cuboid is equal to the sum of the areas of its six rectangular faces.

Surface area of the cuboid

Consider a cuboid whose dimensions are $l \times b \times h$, respectively.



Cuboid with length l , breadth b and height h

The total surface area of the cuboid (TSA) = Sum of the areas of all its six faces

$$\text{TSA (cuboid)} = 2(l \times b) + 2(b \times h) + 2(l \times h) = 2(lb + bh + lh)$$

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.

The lateral surface area of the cuboid = Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC

$$\text{LSA (cuboid)} = 2(b \times h) + 2(l \times h) = 2h(l + b)$$

$$\text{Length of diagonal of a cuboid} = \sqrt{l^2 + b^2 + h^2}$$

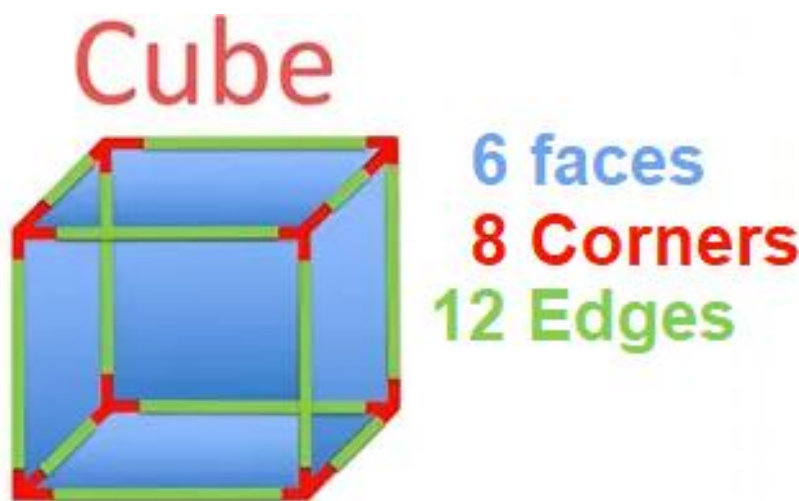
4. If the length of each edge of a **cube** is 'a' units, then:

$$\text{Lateral surface area} = 4 \times (\text{edge})^2$$

$$\text{Total surface area} = 6 \times (\text{edge})^2$$

$$\text{Volume} = (\text{edge})^3$$

$$\text{Diagonal of a cube} = \sqrt{3} \times \text{edge}$$

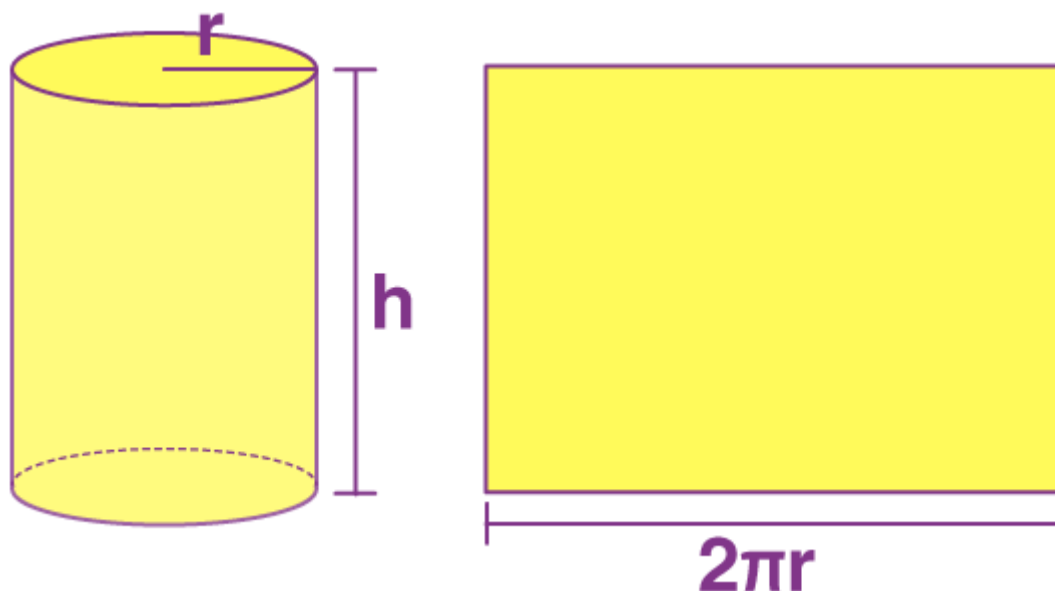


Surface Area and Volume of Cylinder

A cylinder is a solid shape that has two circular bases, connected with each other, through a lateral surface. Thus, there are three faces, two circular and one lateral, of a cylinder. Based on these dimensions, we can find the surface area and volume of a cylinder.

Surface Area of Cylinder

Take a cylinder of base radius r and height h units. The curved surface of this cylinder, if opened along the diameter ($d = 2r$) of the circular base can be transformed into a rectangle of length $2\pi r$ and height h units. Thus,



Transformation of a Cylinder into a rectangle.

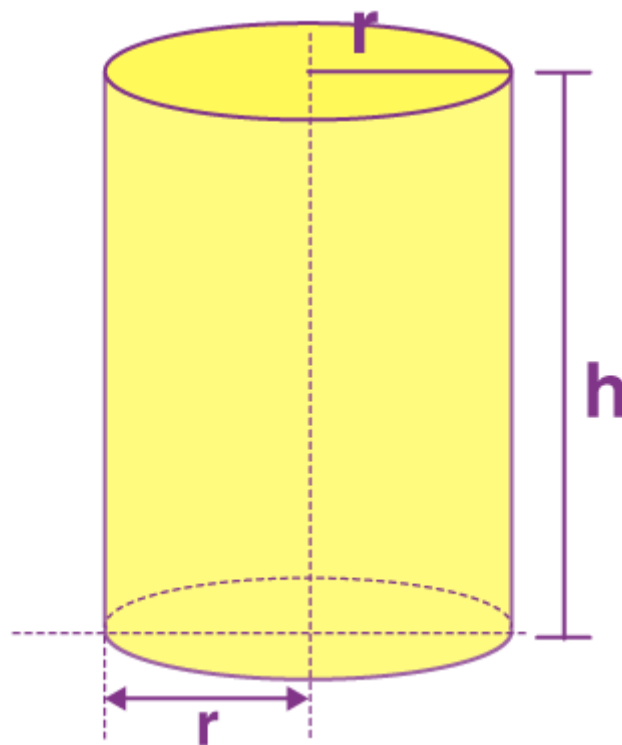
CSA of a cylinder of base radius r and height $h = 2\pi \times r \times h$

TSA of a cylinder of base radius r and height $h = 2\pi \times r \times h + \text{area of two circular bases}$
 $= 2\pi \times r \times h + 2\pi r^2$

$= 2\pi r (h + r)$

Volume of a Cylinder

Volume of a cylinder = Base area \times height $= (\pi r^2) \times h = \pi r^2 h$



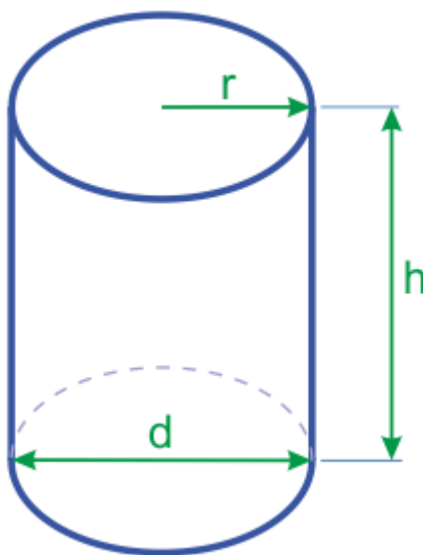
Cylinder with height h and base radius r

5. If r and h respectively denote the radius of the base and the height of a **right circular cylinder**, then: Area of each end or Base area $= \pi r^2$

Area of curved surface or lateral surface area = perimeter of the base \times height = $2\pi rh$

Total surface area (including both ends) = $2\pi rh + \pi r^2 + 2\pi r(h + r)$

Volume = Area of the base \times height = $\pi r^2 h$



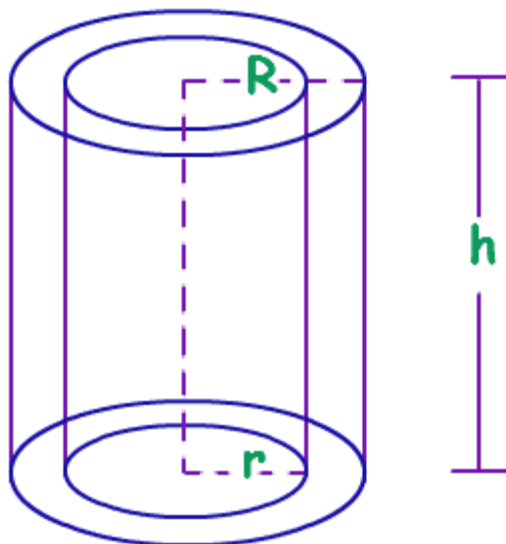
6. If R and r respectively denote the external and internal radii of a **right circular hollow cylinder** and h denotes its height, then:

Area of each end = $\pi R^2 - \pi r^2$

Area of curved surface = $2\pi (R + r)h$

Total surface area = (Area of curved surface) + 2(Area of each end)

= $2\pi (R + r)h + (\pi R^2 - \pi r^2)$



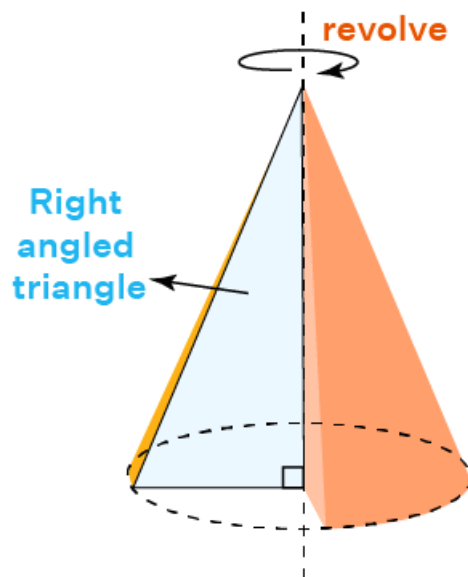
7. If r , h and l respectively denote the radius, height and slant height of a **right circular cone**, then:

Slant height (ℓ) = $\sqrt{h^2 + r^2}$

Area of curved surface = $\pi r \ell = \pi r \sqrt{h^2 + r^2}$

Total surface area = Area of curved surface + Area of base = $\pi r \ell + \pi r^2 = \pi r(\ell + r)$

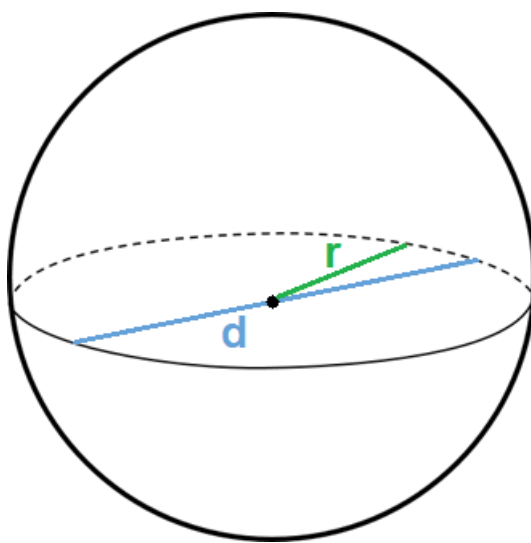
$$\text{Volume} = \frac{1}{3}\pi r^2 h$$



8. If r is the radius of a **sphere**, then:

$$\text{Surface area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$



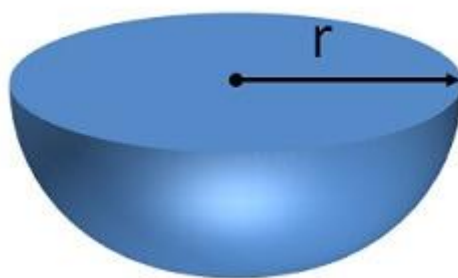
9. If r is the radius of a **hemisphere**, then:

$$\text{Area of curved surface} = 2\pi r^2$$

$$\text{Total surface Area} = \text{Area of curved surface} + \text{Area of base}$$

$$= 2\pi r^2 + \pi r^2 = 3\pi r^2$$

$$\text{Volume} = \frac{2}{3}\pi r^3$$



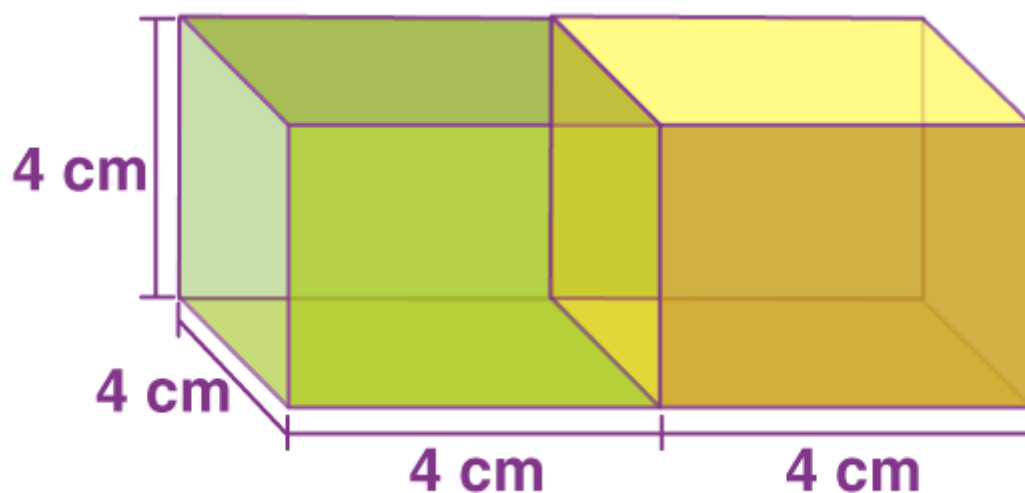
Surface Area of Combined Figures

Areas of complex figures can be broken down and analysed as simpler known shapes. By finding the areas of these known shapes, we can find out the required area of the unknown figure.

Example: 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Length of each cube = $64^{(1/3)} = 4 \text{ cm}$

Since these cubes are joined adjacently, they form a cuboid whose length $l = 8 \text{ cm}$. But height and breadth will remain the same = 4 cm .



Combination of 2 equal cubes

\therefore The new surface area, $TSA = 2(lb + bh + lh)$

$$TSA = 2(8 \times 4 + 4 \times 4 + 8 \times 4)$$

$$= 2(32 + 16 + 32)$$

$$= 2(80)$$

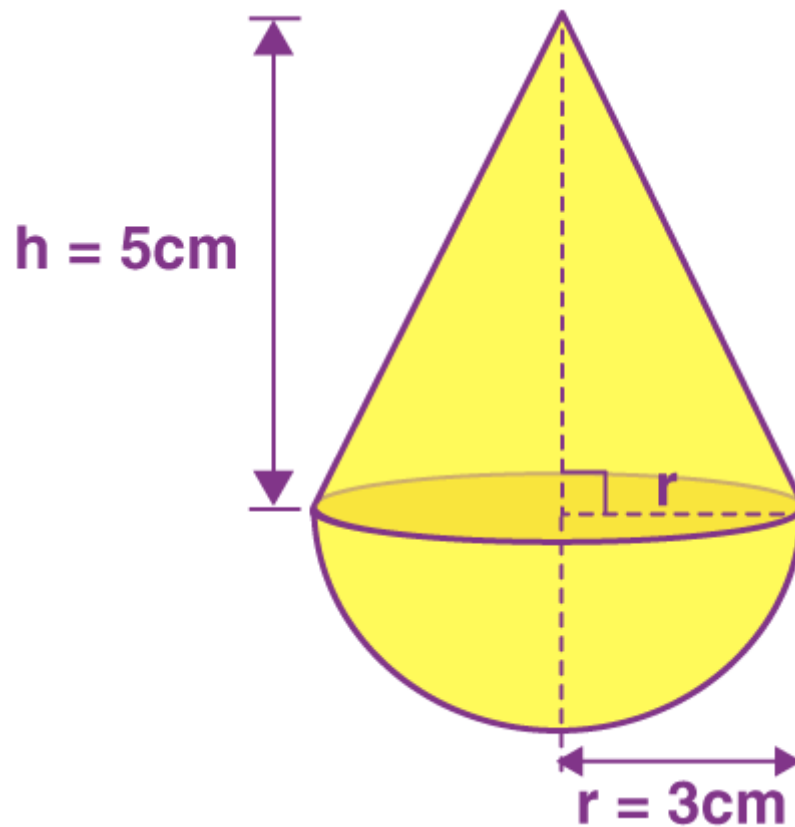
$$TSA = 160 \text{ cm}^2$$

Volume of Combined Solids

The volume of complex objects can be simplified by visualising them as a combination of shapes of known solids.

Example: A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 3 cm and the height of the cone is equal to 5 cm .

This can be visualised as follows:



Volume of combined solids

$$V(\text{solid}) = V(\text{Cone}) + V(\text{hemisphere})$$

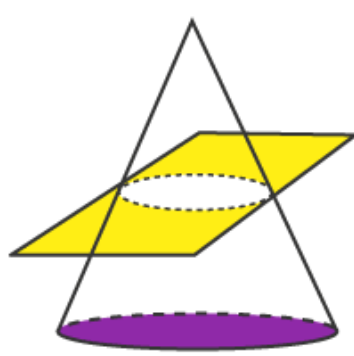
$$V(\text{solid}) = \left(\frac{1}{3}\right)\pi r^2 h + \left(\frac{2}{3}\right)\pi r^3$$

$$V(\text{solid}) = \left(\frac{1}{3}\right)\pi(9)(5) + \left(\frac{2}{3}\right)\pi(27)$$

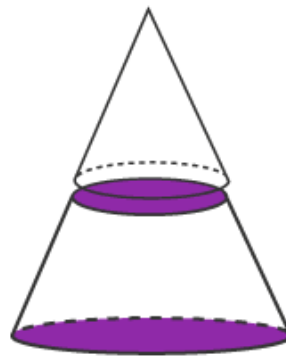
$$V(\text{solid}) = 33\pi \text{ cm}^3$$

Surface area and volume of Frustum of a cone

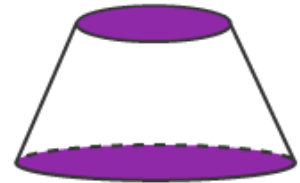
When a solid is cut by a plane, then another form of solids is formed. One such form of solid is the frustum of a cone, which is formed when a plane cuts a cone parallelly to the base of the cone. Let us discuss its surface area and volume here.



Plane parallel
to base cuts
the cone



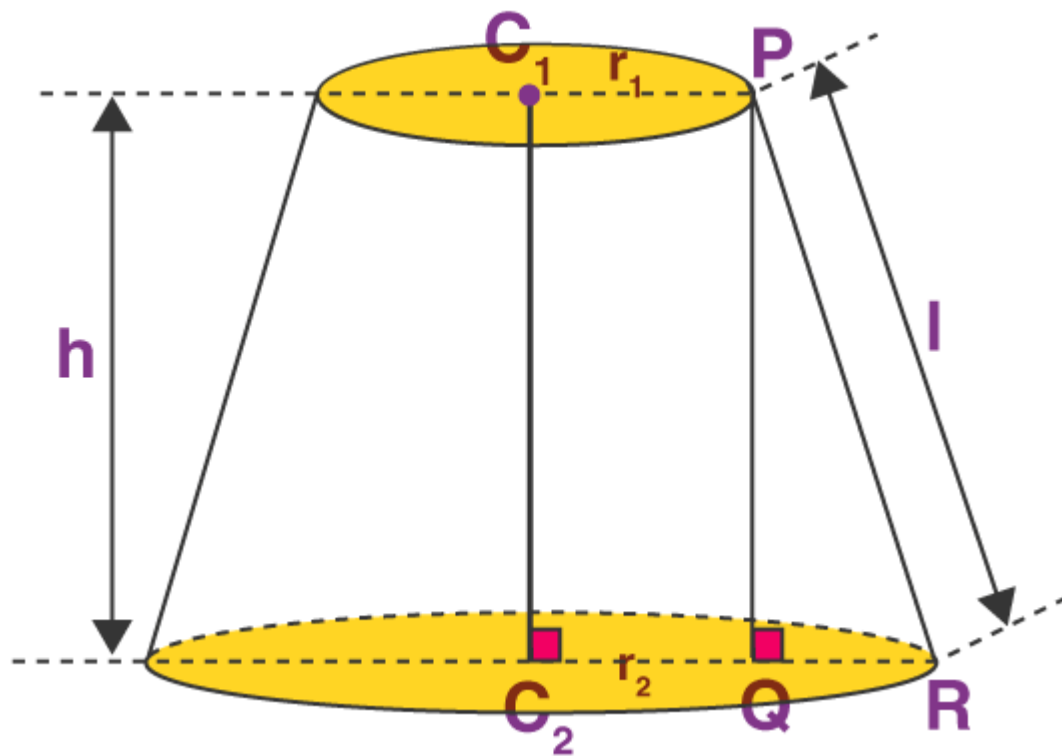
Cone as two
separate parts



Frustum of
a cone

Frustum of cone

If a right circular cone is sliced by a plane parallel to its base, then the part with the two circular bases is called a Frustum.



Frustum with radius r_1 and r_2 and height h

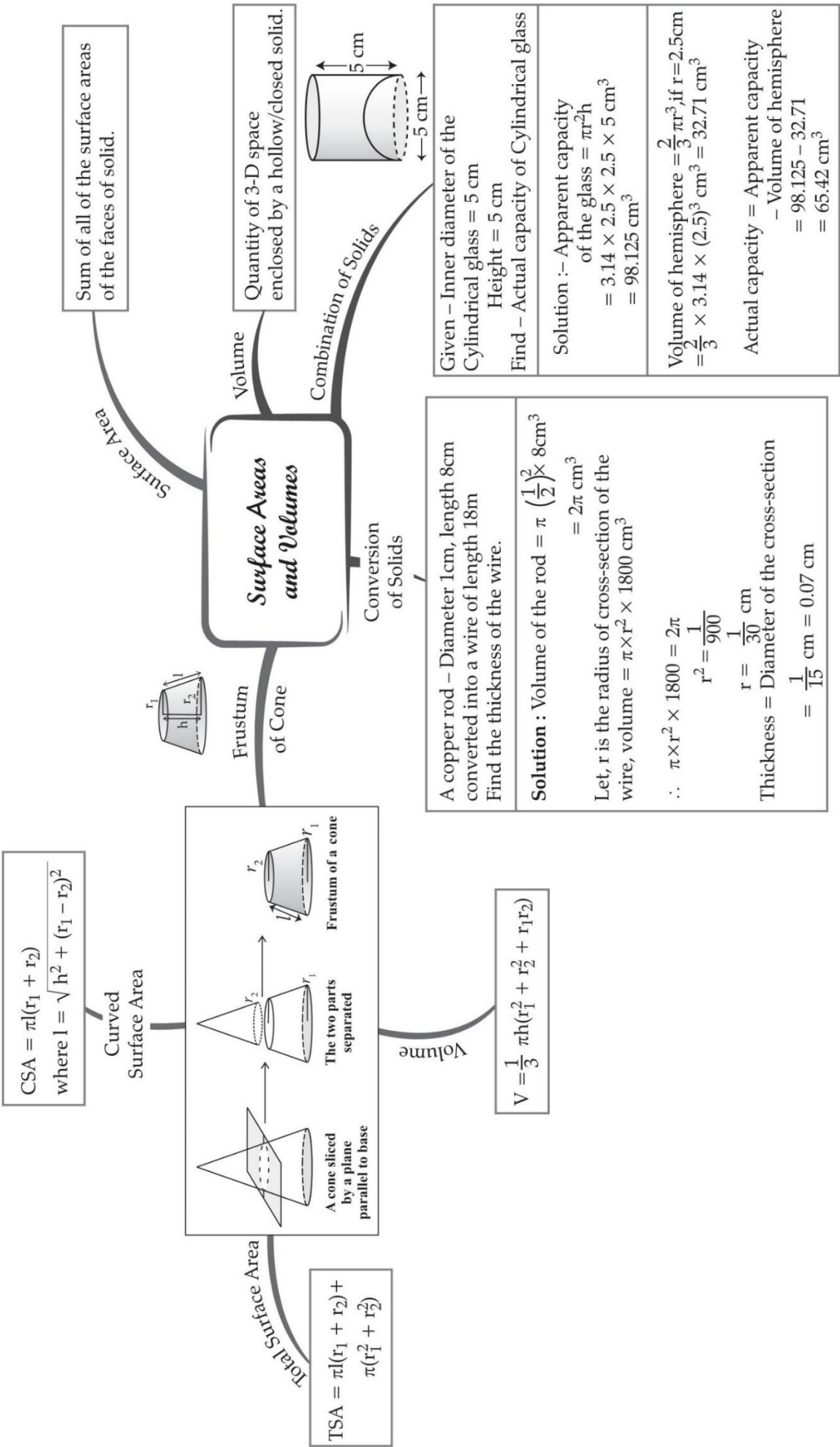
CSA of frustum = $\pi(r_1 + r_2)l$, where $l = \sqrt{h^2 + (r_2 - r_1)^2}$

TSA of the frustum is the CSA + the areas of the two circular faces = $\pi(r_1 + r_2)l + \pi(r_1^2 + r_2^2)$

Volume of a Frustum

The volume of a frustum of a cone = $(1/3)\pi h(r_1^2 + r_2^2 + r_1r_2)$

MIND MAP : LEARNING MADE SIMPLE



Important Questions

Multiple Choice questions-

1. If the surface areas of two spheres are in ratio 16 : 9, then their volumes will be in the ratio:

- (a) 27 : 64
- (b) 64 : 27
- (c) 4 : 3
- (d) 3 : 4

2. A cylinder, a cone and a hemisphere are of equal base and have the same height. What is the ratio of their volumes?

- (a) 3 : 1 : 2
- (b) 3 : 2 : 1
- (c) 1 : 2 : 3
- (d) 1 : 3 : 2

3. If the area of three adjacent faces of cuboid are X, Y and Z respectively, then the volume of cuboid is:

- (a) XYZ
- (b) 3XYZ
- (c) \sqrt{xyz}
- (d) $\sqrt{3xyz}$

4. The volumes of two spheres are in the ratio 27 : 8. The ratio of their curved surface is:

- (a) 9 : 4
- (b) 4 : 9
- (c) 3 : 2
- (d) 2 : 3

5. The ratio of the volumes of two spheres is 8 : 27. If r and R are the radii of spheres

respectively, then $(R - r) : r$ is:

- (a) 1 : 2
- (b) 1 : 3
- (c) 2 : 3
- (d) 4 : 9

6. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is:

- (a) 27 : 20
- (b) 20 : 27
- (c) 9 : 4
- (d) 4 : 9

7. If the radius of base of a right circular cylinder is halved, keeping the height same, the ratio of the volume of the reduced cylinder to that of the original cylinder is:

- (a) 2 : 3
- (b) 3 : 4
- (c) 1 : 4
- (d) 4 : 1

8. If the volumes of a cube is 1728 cm^3 , the length of its edge is equal to:

- (a) 7 cm
- (b) 12 cm
- (c) 18 cm
- (d) 19 cm

9. The volume (in cm^3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm is: .

- (a) 9.7
- (b) 72.6
- (c) 58.2

(d) 19.4

10. The circumference of the edge of hemispherical bowl is 132 cm. When π is taken as $\frac{22}{7}$, the capacity of bowl in cm^3 is:

(a) 2772

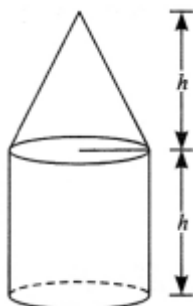
(b) 924

(c) 19404

(d) 9702

Very Short Questions:

1. What is the capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom?
2. A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone. What is the total surface area of the combined solid?



3. Two identical solid hemispheres of equal base radius r cm are struck together along their bases. What will be the total surface area of the combination?
4. A solid ball is exactly fitted inside the cubical box of side a . What is the volume of the ball?
5. If two cubes of edge 5 cm each are joined end to end, find the surface area of the resulting cuboid.
6. A solid piece of iron in the form of a cuboid of dimension $49 \text{ cm} \times 33 \text{ cm} \times 24 \text{ cm}$ is melted to form a solid sphere. Find the radius of sphere.
7. A mason constructs a wall of dimensions $270 \text{ cm} \times 300 \text{ cm} \times 350 \text{ cm}$ with the bricks each of size $22.5 \text{ cm} \times 11.25 \text{ cm} \times 8.75 \text{ cm}$ and it is assumed that space is covered by the mortar. Find the number of bricks used to construct the wall.

8. The radii of the ends of a frustum of a cone 40 cm high are 20 cm and 11 cm. Find its slant height.
9. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?
10. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes?

Short Questions :

1. What is the ratio of the volume of a cube to that of a sphere which will fit inside it?
2. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, find the height of the frustum.
3. If the slant height of the frustum of a cone is 10 cm and the perimeters of its circular base are 18 cm and 28 cm respectively. What is the curved surface area of the frustum?
4. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.
5. A vessel is in the form of a hollow hemisphere mounted by a hollow 7 cm cylinder. The diameter of the hemisphere is 14 cm and the total height T of the vessel is 13 cm. Find the inner surface area of the vessel.
6. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.
7. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.
8. The dimensions of a solid iron cuboid are $4.4 \text{ m} \times 2.6 \text{ m} \times 1.0 \text{ m}$. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.
9. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

OR

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

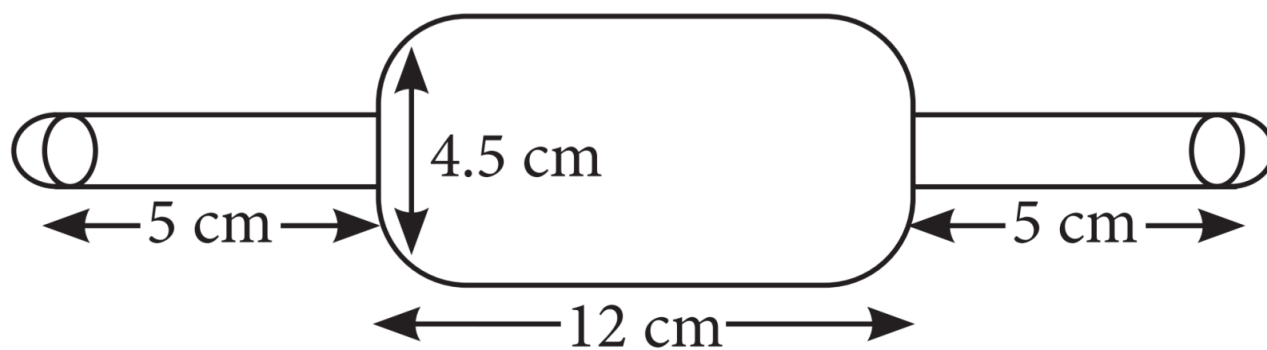
10. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Long Questions :

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid. (Use $\pi = \frac{22}{7}$)
2. A hemispherical tank, full of water, is emptied by a pipe at the rate of $\frac{25}{7}$ litres per sec. How much time will it take to empty half the tank if the diameter of the base of the tank is 3 m?
3. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.
4. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.
5. From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = \frac{22}{7}$)

Case Study Questions:

1. Arpana is studying in X standard. While helping her mother in kitchen, she saw rolling pin made of steel and empty from inner side, with two small hemispherical ends as shown in the figure.



- Find the curved surface area of two identical cylindrical parts, if the diameter is 2.5 cm and length of each part is 5 cm.
 - 475 cm²

b. 78.57cm^2

c. 877cm^2

d. 259.19cm^2

ii. Find the volume of big cylindrical part.

190.93cm^3

75cm^3

77cm^3

83.5cm^3

iii. Volume of two hemispherical ends having diameter 2.5cm, is:

4.75cm^3

8.18cm^3

2.76cm^3

75cm^3

iv. Curved surface area of two hemispherical ends, is:

17.5cm^2

7.9cm^2

19.64cm^2

15.5cm^2

v. Find the difference of volumes of bigger cylindrical part and total volume of the two small hemispherical ends.

175.50cm^3

182.75cm^3

76.85cm^3

96cm^3

2. Isha's father brought an ice-cream brick, empty cones and scoop to pour the ice-cream into cones for all the family members. Dimensions of the ice-cream brick are $(30 \times 25 \times 10)\text{cm}^3$ and radius of hemi-spherical scoop is 3.5cm. Also, the radius and height of cone are 3.5cm and 15cm respectively.



i. The quantity of ice-cream in the brick (in litres) is:

- a. 3
- b. 7.5
- c. 2.5
- d. 4.5

ii. Volume of hemispherical scoop is:

- a. 40.6cm^3
- b. 2509cm^3
- c. 89.83cm^3
- d. 20cm^3

iii. Volume of a cone is:

- a. 148cm^3
- b. 250.05cm^3
- c. 145.83cm^3
- d. 192.5cm^3

iv. The minimum number of scoops required to fill one cone up to brim is:

- a. 2
- b. 3
- c. 4

d. 5

v. The number of cones that can be filled up to brim using the whole brick is:

a. 15

b. 39

c. 40

d. 42

Assertion Reason Questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true and R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) Both A and R is false.

Assertion: If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75%.

Reason: Curved surface area is increased when diameter decreases

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true and R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) Both A and R is false.

Assertion: The external dimensions of a wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of the wood is 15 mm, then the internal volume is 765 cm^3 .

Reason: If external dimensions of a rectangular box be l , b and h and the thickness of its sides be x , then its internal volume is $(l - 2x)(b - 2x)(h - 2x)$.

Answer Key-

Multiple Choice questions-

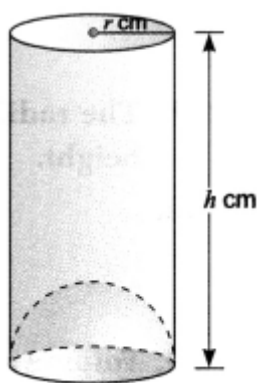
1. (b) 64 : 27

2. (a) 3 : 1 : 2

3. (c) \sqrt{xyz}
4. (a) 9 : 4
5. (a) 1 : 2
6. (b) 20 : 27
7. (c) 1 : 4
8. (b) 12 cm
9. (d) 19.
10. (a) 2772

Very Short Answer :

1.



Capacity of the given vessel

= capacity of cylinder – capacity of hemisphere

$$= \pi r^2 h - \frac{2}{3} \pi r^3 = \frac{\pi r^2}{3} (3h - 2r)$$

2. The total surface area of the combined solid in Fig.

= curved surface area of cone + curved surface area of cylinder + area of the base.

$$= \pi r l + 2\pi r h + \pi r^2 = \pi r(l + 2h + r) = \pi r(\sqrt{r^2 + h^2} + 2h + r)$$

3. The resultant solid will be a sphere of radius r whose total surface area is $4\pi r^2$.
4. Diameter of the solid ball = edge of the cube = a

$$\therefore \text{Volume of the ball} = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{4}{3} \times \frac{1}{8}\pi a^3 = \frac{1}{6}\pi a^3$$

5. Total length (l) = 5 + 5 = 10 cm

Breadth (b) = 5 cm, Height (h) = 5 cm

Surface Area = 2 (lb + bh + lh)

$$= 2(10 \times 5 + 5 \times 5 + 5 \times 10) = 2 \times 125 = 250 \text{ cm}^2$$

6. Volume of iron piece = Volume of the sphere formed

$$= 49 \times 33 \times 24 = \frac{4}{3}\pi r^2$$

$$r^3 = \frac{49 \times 33 \times 24 \times 3 \times 7}{4 \times 22}$$

$$r = 21 \text{ cm}$$

7. Space occupied with bricks = $\frac{7}{8} \times$ volume of the wall

$$= \frac{7}{8} \times 270 \times 300 \times 350$$

$$\therefore \text{Number of bricks} = \frac{\text{Space occupied with bricks}}{\text{Volume of one brick}} = \frac{\frac{7}{8} \times 270 \times 300 \times 350}{22.5 \times 11.25 \times 8.75} = 11,200$$

- 8.

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{40^2 + (20 - 11)^2} = \sqrt{1600 + 81} = \sqrt{1681} = 41 \text{ cm}$$

9. As per question

Volume of hemisphere = Surface area of hemisphere

$$= \frac{2}{3}\pi r^2 = 3\pi r^2, \text{ units } r = \frac{9}{2} \text{ units.}$$

10. Volume of a cone: Volume of a hemisphere: Volume of a cylinder

$$= \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$$

$$= \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 \quad (\because r = h)$$

$$= 1 : 2 : 3$$

Short Answer :

1. Let edge of the cube be 'a'.

Then, diameter of the sphere that will fit inside the given cube = a

∴ Volume of the cube : Volume of the sphere.

$$= a^3 : \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = a^3 : \frac{4}{3} \times \frac{1}{8}\pi a^3 = a^3 : \frac{1}{6}\pi a^3 = 6 : \pi$$

2. Let r and R be radii of the circular ends of the frustum of the cone.

Then, $R - r = 4$, $l = 5$

We know, $l^2 = (R - r)^2 + h^2$

$$\Rightarrow 5^2 = 4^2 + h^2 \text{ or } h^2 = 25 - 16 = 9$$

$$\Rightarrow h = 3 \text{ cm}$$

3. Let r and R be the radii of the two circular ends of the frustum of the cone.

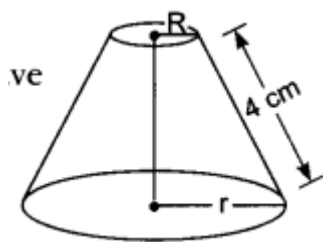
Then, $2\pi r = 18$ and $2\pi R = 28$

$$\Rightarrow r = \frac{18}{2\pi} \quad \text{and} \quad R = \frac{28}{2\pi} \quad \Rightarrow \quad r = \frac{9}{\pi} \quad \text{and} \quad R = \frac{14}{\pi}$$

Now, curved surface area of the frustum = $\pi(r + R)l$

$$= \pi\left(\frac{9}{\pi} + \frac{14}{\pi}\right) \times 10 = 23 \times 10 = 230 \text{ cm}^2$$

- 4.



We have, slant height, $l = 4 \text{ cm}$

Let R and r be the radii of two circular ends respectively. Therefore, we have

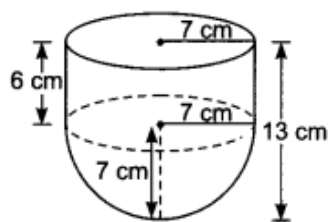
$$\Rightarrow 2\pi R = 18 = \pi R = 9$$

$$\Rightarrow 2\pi r = 6 = \pi r = 3$$

∴ Curved surface area of the frustum = $(\pi R + \pi r)l$

$$= (9 + 3) \times 4 = 12 \times 4 = 48 \text{ cm}^2$$

5.



Here, radius of hemisphere = radius of cylinder = r cm = 7 cm

and height of cylinder, $h = (13 - 7)$ cm = 6 cm

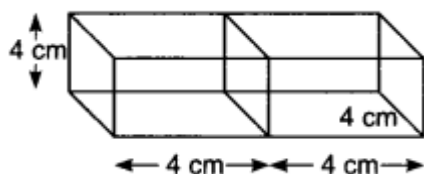
Now, inner surface area of the vessel

= Curved surface area of the cylindrical part + Curved surface area of hemispherical part = $(2\pi rh + 2\pi r^2) = 2\pi r (h + r)$

$$= 2 \times \frac{22}{7} \times 7 (6 + 7)$$

$$= 2 \times 22 \times 13 = 572 \text{ cm}^2$$

6.



Let the length of each edge of the cube of volume 64 cm^3 be x cm.

Then, Volume = 64 cm^3

$$\Rightarrow x^3 = 64$$

$$\Rightarrow x^2 = 43$$

$$\Rightarrow x = 4 \text{ cm}$$

4 cm The dimensions of cuboid so formed are

$$l = \text{Length} = (4 + 4) \text{ cm} = 8 \text{ cm}$$

$$b = \text{Breadth} = 4 \text{ cm and } h = \text{Height} = 4 \text{ cm}$$

$$\therefore \text{Surface area of the cuboid} = 2 (lb + bh + lh)$$

$$= 2 (8 \times 4 + 4 \times 4 + 8 \times 4)$$

$$= 2 (32 + 16 + 32)$$

$$= 160 \text{ cm}^2$$

7. The greatest diameter that a hemisphere can have = 7 cm = l

$$\text{Radius of the hemisphere (R)} = \frac{7}{2} \text{ cm}$$

∴ Surface area of the solid after surmounting hemisphere

$$= 6l^2 - \pi R^2 + 2\pi R^2 = 6l^2 + \pi R^2$$

$$= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 6 \times 49 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 294 + 38.5 = 332.5 \text{ cm}^2$$

8. Let the length of pipe by h m.

$$\text{Volume of cuboid} = 4.4 \times 2.6 \times 1 \text{ m}^2$$

Inner and outer radii of cylindrical pipe are 30 cm, (30 + 5) cm = 35 cm

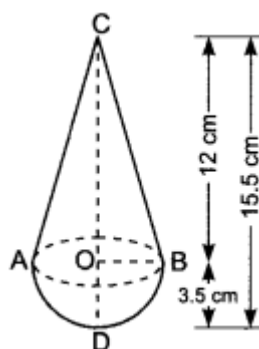
$$\begin{aligned} \therefore \text{Volume of material used} &= \frac{\pi}{100^2} (35^2 - 30^2) \times h \text{ m}^3 \\ &= \frac{\pi}{100^2} \times 65 \times 5h \quad [\text{using } a^2 - b^2 = (a + b)(a - b)] \end{aligned}$$

$$\text{Now } \frac{\pi}{100^2} \times 65 \times 5h = 4.4 \times 2.6$$

$$\Rightarrow h = \frac{7 \times 4.4 \times 2.6 \times 100 \times 100}{22 \times 65 \times 5}$$

$$\Rightarrow h = 112 \text{ m}$$

- 9.



We have,

$$CD = 15.5 \text{ cm and } OB = OD = 3.5 \text{ cm}$$

Let r be the radius of the base of cone and h be the height of conical part of the toy.

Then, $r = OB = 3.5$ cm

$h = OC = CD - OD = (15.5 - 3.5)$ cm = 12 cm

$$\begin{aligned} l &= \sqrt{r^2 + h^2} = \sqrt{3.5^2 + 12^2} \\ &= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5 \text{ cm} \end{aligned}$$

Also, radius of the hemisphere, $r = 3.5$ cm

$$\begin{aligned} \therefore \text{Total surface area of the toy} &= \text{Surface area of cone} + \text{Surface area of hemisphere} \\ &= \pi r l + 2\pi r^2 = \pi r(l + 2r) = \frac{22}{7} \times 3.5 (12.5 + 2 \times 3.5) \\ &= \frac{22}{7} \times 3.5 \times 19.5 = 214.5 \text{ cm}^2 \end{aligned}$$

10. Here, we have

Edge of the cube = l = Diameter of the hemisphere

Therefore, radius of the hemisphere = $\frac{l}{2}$

\therefore Surface area of the remaining solid after cutting out the hemispherical

$$\begin{aligned} \text{depression} &= 6l^2 - \pi \left(\frac{l}{2}\right)^2 + 2\pi \left(\frac{l}{2}\right)^2 \\ &= 6l^2 + \pi \times \frac{l^2}{4} = \frac{l^2}{4} (24 + \pi) \end{aligned}$$

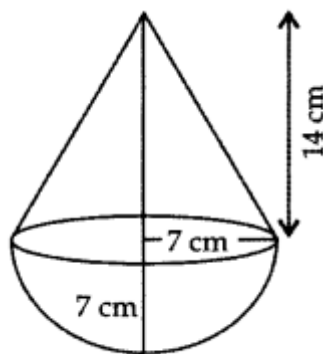
Long Answer :

1. Radius, $r = 7$ cm

Height of cone, $h = 2(7) = 14$ cm

Volume of solid = Vol. of hemisphere + Volume of cone

$$\begin{aligned}
 &= \frac{2}{3} \pi r^3 + \frac{1}{3} \pi r^2 h \\
 &= \frac{1}{3} \pi r^2 (2r + h) \\
 &= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 (2(7) + 14) \\
 &= \frac{22 \times 7}{3} \times 28 = \frac{4312}{3} \\
 &= 1437.\bar{3} \text{ cm}^3
 \end{aligned}$$



2.

Here, $r = \frac{3}{2} \text{ m}$

$$\frac{25}{7} \text{ lt} = \frac{1}{1000} \times \frac{25}{7} \text{ m}^3 = \frac{1}{280} \text{ m}^3 \dots [\because 1 \text{ lt.} = \frac{1}{1000} \text{ m}^3]$$

$$\text{Required time} = \frac{\frac{1}{2} \text{ Vol. of hemispherical tank}}{\text{Vol. of pipe}}$$

$$\begin{aligned}
 &= \frac{\frac{1}{2} \times \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}}{\frac{1}{280}} \\
 &= \frac{22 \times 9}{7 \times 8} \times \frac{280}{1} = 990 \text{ secs.} \\
 &= 16 \text{ mins. } 30 \text{ secs.}
 \end{aligned}$$

\therefore Required time is $16\frac{1}{2}$ mins.

3. Radius of tank, $r_1 = 40 \text{ cm}$

Internal radius of cylindrical pipe, $r_2 = \frac{2}{2} = 1 \text{ cm}$

Let the height of rises water, $h_1 = ?$

Length of water flow in 1 second = 0.4 m

$$= \frac{4}{10} \times 100 = 40 \text{ cm}$$

\therefore Length of water flow in 30 minutes, h_2

$$= 40 \times 60 \times 30 = 72,000 \text{ cm}$$

Volume of water in cylinder tank

= Volume of water flow from cylindrical pipe in half an hour

$$\begin{aligned}\text{As } \pi r_1^2 h_1 &= \pi r_2^2 h_2 \\ \therefore 40 \times 40 \times h_1 &= 1 \times 1 \times 72,000 \\ h_1 &= \frac{72,000}{40 \times 40} = 45 \text{ cm}\end{aligned}$$

\therefore Level of water in cylinder tank rises in half an hour, $h_1 = 45$ cm

4.

$$\text{Radius of a marble, } r = \frac{1.4}{2} = \frac{7}{10} \text{ cm,}$$

$$\text{Radius of cylinder, } R = \frac{7}{2} = 3.5 \text{ cm}$$

No. of spherical marbles

$$\begin{aligned}&= \frac{\text{Vol. of water rise in cylinder}}{\text{Vol. of one marble (sphere)}} \\ 150 &= \frac{\pi \left(\frac{7}{2} \times \frac{7}{2} \times h \right)}{\frac{4}{3} \times \pi \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}} \\ &\quad \dots \left[\begin{array}{l} \text{Vol. of cylinder} = \pi r^2 h \\ \text{Vol. of sphere} = \frac{4}{3} \pi r^3 \end{array} \right. \\ \Rightarrow \frac{7}{2} \times \frac{7}{2} \times h &= 150 \times \frac{4}{3} \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} \\ h &= \frac{50 \times 2 \times 2 \times 4 \times 7}{10 \times 10 \times 10} = \frac{56}{10} \text{ cm} \\ \therefore \text{ Rise in water level, } h &= \frac{56}{10} = 5.6 \text{ cm}\end{aligned}$$

5.

$$\text{Given: } r = \frac{4.2}{2} = 2.1 \text{ cm, } h = 2.8 \text{ cm}$$

$$\begin{aligned}l &= \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (2.8)^2} \\ &= \sqrt{4.41 + 7.84} = \sqrt{12.25} \\ &= 3.5 \text{ cm}\end{aligned}$$

T.S. area of the remaining solid

= C.S. ar. of cyl. + area of base + C.S. ar. of cone

$$= 2\pi rh + \pi r^2 + \pi rl$$

$$= \pi r(2h + r + l)$$

$$= \frac{22}{7} \times 2.1 (5.6 + 2.1 + 3.5)$$

$$= 22 \times 0.3(11.2)$$

$$= 73.92 \text{ cm}^2$$

Case Study Answer:

1. Answer:

i. (b) 78.57cm^2

Solution:

Curved surface area of two identical cylindrical parts,

$$= 2 \times 2\pi rh = 2 \times 2 \times \frac{22}{7} \times \frac{2.5}{2} \times 5$$

$$= 78.57\text{cm}^2$$

ii. (a) 190.93cm^3

Solution:

Volume of big cylindrical part = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} \times 12190.93\text{cm}^3$$

iii. (b) 8.18cm^3

Solution:

Volume of two hemispherical ends = $2 \times \frac{2}{3} \pi r^3$

$$= \frac{2 \times 2}{3} \times \frac{22}{7} \times \left(\frac{2.5}{2}\right)^3 = 8.18\text{cm}^3$$

iv. (c) 19.64cm^2

Solution:

Curved surface area of two hemispherical ends,

$$= 2 \times 2\pi r^2 = 2 \times 2 \times \frac{22}{7} \times \frac{2.5}{2} \times \frac{2.5}{2} = 19.64\text{cm}^2$$

v. (b) 182.75cm^3

Solution:

Difference of volume of bigger cylinder to two small hemispherical ends = $190.93 - 8.18 = 182.75\text{cm}^3$

2. Answer:

i. (b) 7.5

Solution:

$$\begin{aligned}\text{Quantity of ice-cream in the brick} &= \text{volume of the brick} = (30 \times 25 \times 10)\text{cm}^3 = 7500\text{cm}^3 \\ &= \frac{7500}{1000}l \quad [\because 1l = 1000\text{cm}^3] \\ &= 7.5l\end{aligned}$$

ii. (c) 89.83cm^3

Solution:

$$\begin{aligned}\text{Volume of hemispherical scoop} &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 = \frac{1886.5}{21} = 89.83\text{cm}^3\end{aligned}$$

iii. (d) 192.5cm^3

Solution:

$$\begin{aligned}\text{Volume of cone} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 15 = \frac{4042.5}{21} = 192.5\text{cm}^3\end{aligned}$$

iv. (a) 2

Solution:

Number of scoops required to fill one cone

$$= \frac{\text{Volume of a cone}}{\text{Volume of a scoop}} = \frac{192.5}{89.83} = 2.14 \approx 2$$

v. (b) 39

Solution:

Number of cones that can be filled using the,

$$\begin{aligned}\text{Whole brick} &= \frac{\text{Volume of brick}}{\text{Volume of 1 cone}} \\ &= \frac{7500}{192.5} = 38.960 \approx 39\end{aligned}$$

Assertion Reason Answer-

(c) A is true but R is false.

(a) Both A and R are true and R is the correct explanation of A.